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AUTHOR Sophian, Catherine
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ABSTRACT

This study systematically examined developmental changes in the flexibility of preschool children's counting skills by assessing their use of counting to solve different kinds of quantitative problems at several preschool age levels. Three principal kinds of quantitative problems were considered: (1) quantifying a given set; (2) generating a set of a specified numerosity; and (3) comparing two numerical quantities. In the first experiment 18 three-year-olds and 18 three-and-one-half-year-olds were tested on a relative numerosity task and on counting sets of objects. The second experiment included 17 three-and-one-half-year-olds, 19 four-year-olds, and 18 four-and-one-half-year-olds; they were tested on numerosity and on counting a wider variety of quantitative problems. Recurrent limitations on young children's use of counting were found that cannot be wholly explained either by lack of knowledge about how to count or by performance factors. Young children's limited use of counting may stem from important restrictions on what they understand about when or why they should count. Educational implications are then discussed. (MNS)

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Early Developments in Children's Use of Counting to Solve Quantitative Problems

Catherine Sophian

Carnegie-Mellon University

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Early Developments in Children's Use of Counting to Solve Quantitative Problems

Although there is impressive evidence that preschool children know how to count, their understanding of the ways in which counting can be used to solve quantitative problems may still be precarious. Gelman and Gallistel's (1978) abstraction or what-to-count principle points to at least one dimension along which preschoolers' use of counting is fairly robust: they will count heterogeneous as well as homogeneous sets. Yet other researchers have provided data that seems to indicate restrictions on young children's use of counting: for instance, Shaeffer, Eggleston, & Scott (1974) report that young preschoolers seldom count when asked to "put x candies in the cup", and Michie (1984) reports that they seldom use counting to compare the sets in a number conservation task (see also Siegler, 1981). These findings are important theoretically, because they help to clarify the relationship between the development of numerical understanding and the acquisition of counting skill (c.f., Gelman & Gallistel, 1978; Piaget, 1965). They are also important educationally, because counting provides a crucial foundation for much of the elementary school arithmetic curriculum. Children count on their fingers as they are learning basic arithmetic facts (Siegler & Shrager, 1984), and they count out sets of objects to be combined or compared as they learn to solve simple arithmetic word problems (see Briars & Larkin, 1984; Carpenter & Moser, 1984; Riley, Greeno, & Heller, 1983).

The present research systematically examined developmental changes in the flexibility of preschool children's counting skills by assessing children's use of counting to solve different kinds of quantitative problems at several age levels within the preschool period. Three principal kinds of quantitative

problems were considered: problems requiring the child to quantify a given set; problems requiring the child to generate a set of a specified numerosity; and problems requiring the child to compare two numerical quantities.

The first experiment focussed on young preschoolers' use of counting to compare the numerosities of two sets. Eighteen 3-year-olds and 18 3-1/2-year-olds were tested on a relative numerosity judgment task, and also on a posttest evaluating their ability to accurately count sets of objects when explicitly instructed to do so. On the relative numerosity judgment problems, children were shown two complementary sets of objects, e.g., clowns and balloons, and asked questions of the form: "Are there enough x's (e.g., balloons) so that every y (e.g., clown) can get one?". The numerosity of the y's was either 3, 4, 6, or 7, and the numerosity of the x's was either the same or one less. A control condition, in which this question could be answered without counting, was included to ensure that children understood the questions. For this condition, the objects from the two sets were paired insofar as possible, so that the numerical equivalence of the two sets could be determined simply by seeing whether there was a y that was not paired with an x. In the experimental condition, the two sets were spatially separated and arranged in different geometric configurations so that perceptual comparison was not a practicable strategy.

The data from the relative numerosity judgment problems are summarized in Table 1. Above chance performance in the control condition established that children understood the questions they were asked about relative numerosity. However, children seldom counted in order to answer these questions, even when the sets were spatially separated and so could not be compared directly.

A further analysis took into account children's performance on the

posttest in order to determine whether children's frequent failures to count were due simply to lack of the requisite counting ability. Children were cross-classified according to whether or not they counted on any of the experimental problems and whether or not they counted sets of at least four accurately on the posttest. The post-test was a conservative measure of children's counting abilities for this purpose, since children had to count accurately to be scored as able to count a given numerosity there, whereas on the experimental task they were credited with counting if they made any attempt to count, even if it contained errors. Nevertheless, the data clearly indicated that lack of counting ability, as assessed by the post-test, could not account for children's failures to count on the experimental task. As can be seen in the top half of Figure 1, there were a number of children who could not count even four objects accurately. However, if this were the only source of counting limitations, then all the children who could count on the post-test should also have shown some use of counting on the experimental problems. The only instances of mixed performance (i.e., success on the post-test but no counting on the experimental task, or failure on the post-test but some use of counting on the experimental task) would be due to measurement errors which should be equally likely to occur in both directions. As Figure 1 shows, children were actually much more likely to succeed on the post-test but still not count on the experimental task than vice versa, McNemar's test, $p = .007$. Thus, young children's tendency not to use counting to compare two numerical quantities cannot be attributed solely to a lack of the requisite counting ability.

The second experiment had two main goals. The first was to carry out a broad replication of Experiment 1, with procedural modifications designed to

test some alternative interpretations of the Experiment 1 results. The second was to extend the research to consider children's use of counting on a wider variety of quantitative problems.

In re-examining the limitations on young preschoolers' use of counting to compare two numerical quantities that were found in Experiment 1, two principal changes were made. First, the relative numerosity judgment task was replaced with a more active task, in order to rule out the possibility that children did not count in Experiment 1 simply because they interpreted the task very narrowly and thought they were not supposed to do anything but "just answer". The compare-sets task in Experiment 2 required children to actively construct a second set of the same numerosity as a given set. Second, Experiment 2 included only children aged 3-1/2 or older, so that methodological problems associated with the severe limitations on 3-year-olds' counting ability would not arise. Two older age groups, 4-year-olds, and 4-1/2-year-olds, were also included in the study in order to trace developmental changes in children's use of counting over the later part of the preschool period. The total sample consisted of 17 3-1/2-year-olds, 19 4-year-olds, and 18 4-1/2-year-olds.

As in Experiment 1, a control condition in which the objects could be directly paired provided a check that children understood the instructions for the compare-sets problems. Children's performance here was compared with their performance in an experimental condition that precluded direct perceptual comparison by requiring the child's set to be constructed on a separate tray from the original set. The question of primary interest was whether children would use counting to compare their set to the original set in the experimental condition.

The data from these two conditions are summarized in Table 2. As in Experiment 1, high levels of performance in the control condition established that children understood the questions they were asked. Nevertheless, children seldom counted in order to solve the problems, even when they were precluded from comparing the sets directly.

Possible limitations on children's knowledge of counting were taken into account in this experiment by comparing children's use of counting on the experimental compare-sets task to their use of counting on a "how many" task, which simply asked children how many objects were in a set. This task functioned much like the post-test in Experiment 1, since if children counted here but not on the experimental compare-sets task, it could be inferred that lack of counting ability was not a sufficient explanation for their failures to use counting to compare two spatially-separated sets. In fact, every child in the study counted (although not necessarily coorectly) on one or more of the "how many" problems, whereas 13 3-1/2-year-olds (76%) and 16 older children (43%) did not count on any of the experimental problems, McNemar's tests, ps < .001. Thus, as in Experiment 1, children's tendency not to use counting to compare sets in this study cannot be due only to lack of counting ability.

Two additional kinds of tasks were included in Experiment 2 to further explore the sources of limitations on children's use of counting. One hypothesis was that children had trouble with the compare-sets task because of its complex goal structure, which required coordinating two separate counts to carry out a comparison. The compare-sets task in Experiment 2 can be decomposed into two subtasks: quantifying the given set, which is essentially the same as the "how many" task, and counting out a new set with that number

of objects. A "put n" task, much like the one used by Schaeffer et al. (1974), corresponded to just this latter part of the compare-sets task. Thus, it provided another test of children's use of counting to solve quantitative problems, without entailing the complex goal structure of the compare-sets task as a whole. As can be seen in Figure 2, children were not much more likely to count on this task than on the compare-sets task, and in particular the 3-1/2-year-olds were far less likely to count here than on the how-many task, McNemar's test, $p = .001$.

Another hypothesis was that children might have trouble keeping in mind a target number while counting a set, as both the compare-sets tasks and the put-n tasks required. In order to assess this possibility, children were also tested on a "count-up" task, in which they had to count to a target number, without any reference to objects. As Figure 3 shows, children were nearly as likely to count on this task as on the "how many" task. Like the how-many task, the count-up task elicited significantly more counting from 3-1/2-year-olds than did either the compare-sets task or the put-n task, McNemar's tests, $ps \leq .008$. However, the 3-1/2-year-olds stopped counting at the number they had been told to count up to only 70% of the time on the average. While this level of performance is high enough to indicate some ability to remember the target number while counting, it does leave open the possibility that some degree of interference was also operating.

In summary, then, the data from Experiments 1 and 2 indicate recurrent limitations on young children's use of counting that cannot be wholly explained either by lack of knowledge about how to count or by such performance factors as misunderstandings of the tasks presented to children, difficulties incorporating counting into a complex goal structure, or

inability to remember a target number while counting. While some of these factors may contribute to children's difficulties, none of them are sufficient to explain the differences we have found in children's use of counting across tasks. These findings strengthen the possibility that young children's limited use of counting may stem from important restrictions on what they understand about when or why they should count.

At least two obvious candidates for how children might decide when to count can be identified. One possibility is that counting is initially viewed as a response to "How many" questions, and its relevance to other quantitative problems is not understood. A second possibility, by no means incompatible with the first, is that children initially think counting is only applicable to static sets and not to sets that unfold over time, such as claps of one's hands or objects being put on a tray. Some evidence for this sort of restriction comes from a detailed examination of the form of counting children used when they counted on the compare-sets and put-n tasks in Experiment 2. Fifteen percent of the counts observed on those tasks involved a strategy of putting out a bunch of objects and then counting them rather than counting the objects while putting them into the set.

A recently-completed study provided further evidence that counting is initially restricted to static sets, by looking at children's performance on a new kind of "how many" task, on which children had to count objects that were visible only as as they were moved from one container into another. If the restrictions on children's counting in Experiments 1 and 2 were due solely to the kinds of questions they were asked, then children should have no difficulty counting on this new task, which includes the familiar "how many" question. On the other hand, if children initially think counting is only

applicable to static sets, they should treat this task much like the put-n task, seldom using counting to solve it. In fact, as Figure 4 shows, 3-1/2-year-olds responded to our how-many/move task in much the same way as they did to the put-n task, which we replicated in this study, and very differently than they did to the static how-many task, which we also replicated. Both the put-n task and the how-many/move task elicited significantly less counting than the how-many/static task, McNemar's tests, ps $\leq$ .01.

In conclusion, then, a series of studies has documented strong limitations on young children's use of counting to solve quantitative problems, and one important component in those limitations appears to be a reluctance to count sets that are unfolding over time. This feature of early counting is important theoretically because it suggests that young children who can count reasonably well may still lack a full understanding of the meaning and purpose of counting. It is also important educationally, because the increasingly flexible use of counting that emerges between about 3-1/2 and 4-1/2 years of age is an important foundation for much of the elementary school arithmetic curriculum. Consideration of some specific ways in which counting contributes to arithmetic learning may help to make this connection clear. At least three aspects of elementary school arithmetic that depend on flexible counting abilities can be identified.

First, counting helps children to learn the basic arithmetic facts. An early strategy for answering addition and subtraction problems is to count on one's fingers, and the associations between problems and their answers that are formed in this way become the basis for later performance without counting (Siegler & Shrager, 1984). The use of counting to determine the answers to

addition and subtraction problems, however, depends on being able to count out a set corresponding to a given number as well as being able to count how many objects are in a set that has already been formed. Thus, it depends on moving beyond the restricted use of counting we see in young preschool children to the more flexible counting abilities older preschoolers generally have achieved.

A second contribution of counting appears to lie in helping children to discover fundamental properties of arithmetic operations like the commutativity of addition. With practice at using counting to solve arithmetic problems, preschoolers spontaneously devise more efficient counting strategies, such as the "min" strategy of counting on from the larger addend for the number of steps indicated by the other addend. This new strategy is important because it is one of the first signs of progress toward understanding commutativity that children show (see Groen & Resnick, 1977). In order to discover the min strategy, however, children must have a fairly general understanding of how counting works to guide them in modifying their counting appropriately.

A third important contribution of counting to arithmetic is in the development of skills for solving arithmetic word problems. One of the first strategies for solving these problems is to count out sets to represent the situation described in the problem and to perform appropriate actions on those sets to solve the problem (Carpenter & Moser, 1984). In its most direct form, however, this strategy only works for a small set of problems (e.g., change problems with the result set unknown, but not compare problems or change problems with the start set unknown). In order to progress toward being able to handle a wider range of word problems, children must be able to reason

quite flexibly about the relationship between counts of sets of objects they can act on in meaningful ways and the answers to problems that may not involve any explicit action or that may describe an action sequence that can only be acted out by reversing the temporal sequence of events (Briars & Larkin, 1984).

These observations suggest that children who have difficulty with early arithmetic instruction may not have developed sufficiently flexible counting procedures to support further arithmetic learning. Although most children appear to develop the requisite counting skills through an informal learning process that takes place before they enter school, some children may need explicit instruction to complete this development. In assessing the adequacy of children's counting, it is important to consider not only whether children know how to count but also whether they understand the applicability of counting to different kinds of counting problems. Likewise, instruction in counting as a foundation for later arithmetic instruction should be designed to promote a flexible understanding of counting and how it applies to a wide range of quantitative problems. By ensuring the adequacy of children's counting as a foundation for arithmetic learning when children begin their schooling, educators may be able to forestall learning problems that would otherwise have a cumulative impact on children's progress through elementary school curriculum.

Table 1: Problem-Solving Accuracy and Use of Counting on Relative Numerosity Judgment Problems (Experiment 1)

	Proportion Correct (chance=.50)		Proportion of Children who Count on One or More Problems		
	small numbers	large numbers	small numbers	large numbers	any set size
Control task					
3-year-olds	.79	.61	.11	0	.11
3-1/2-year-olds	.92	.75	0	0	0
Experimental task					
3-year-olds	.76	.53	.06	.06	.11
3-1/2-year-olds	.83	.60	.50	.17	.50

Table 2: Problem-Solving Accuracy and Use of Counting on Constructive Compare-Sets Problems (Experiment 2)

	Proportion Correct		Proportion of Children who Count on One or More Problems		
	small numbers	large numbers	small numbers	large numbers	any set size
Control task					
3-1/2-year-olds	.91	.68	.06	.12	.18
4-year-olds	.87	.90	.16	.16	.26
4-1/2-year-olds	.97	.92	.17	.11	.17
Experimental task					
3-1/2-year-olds	.30	.15	.24	.12	.24
4-year-olds	.53	.19	.53	.42	.63
4-1/2-year-olds	.72	.31	.39	.44	.50

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EXPERIMENTAL PROBLEMS

POST-TEST: largest set correctly counted	never count	count at least once
< 4	7 @ 3 yrs. 1 @ 3 1/2 yrs.	1 @ 3 yrs. 3 @ 3 1/2 yrs.
≥ 4	9 @ 3 yrs. 8 @ 3 1/2 yrs.	1 @ 3 yrs. 6 @ 3 1/2 yrs.

Figure 1

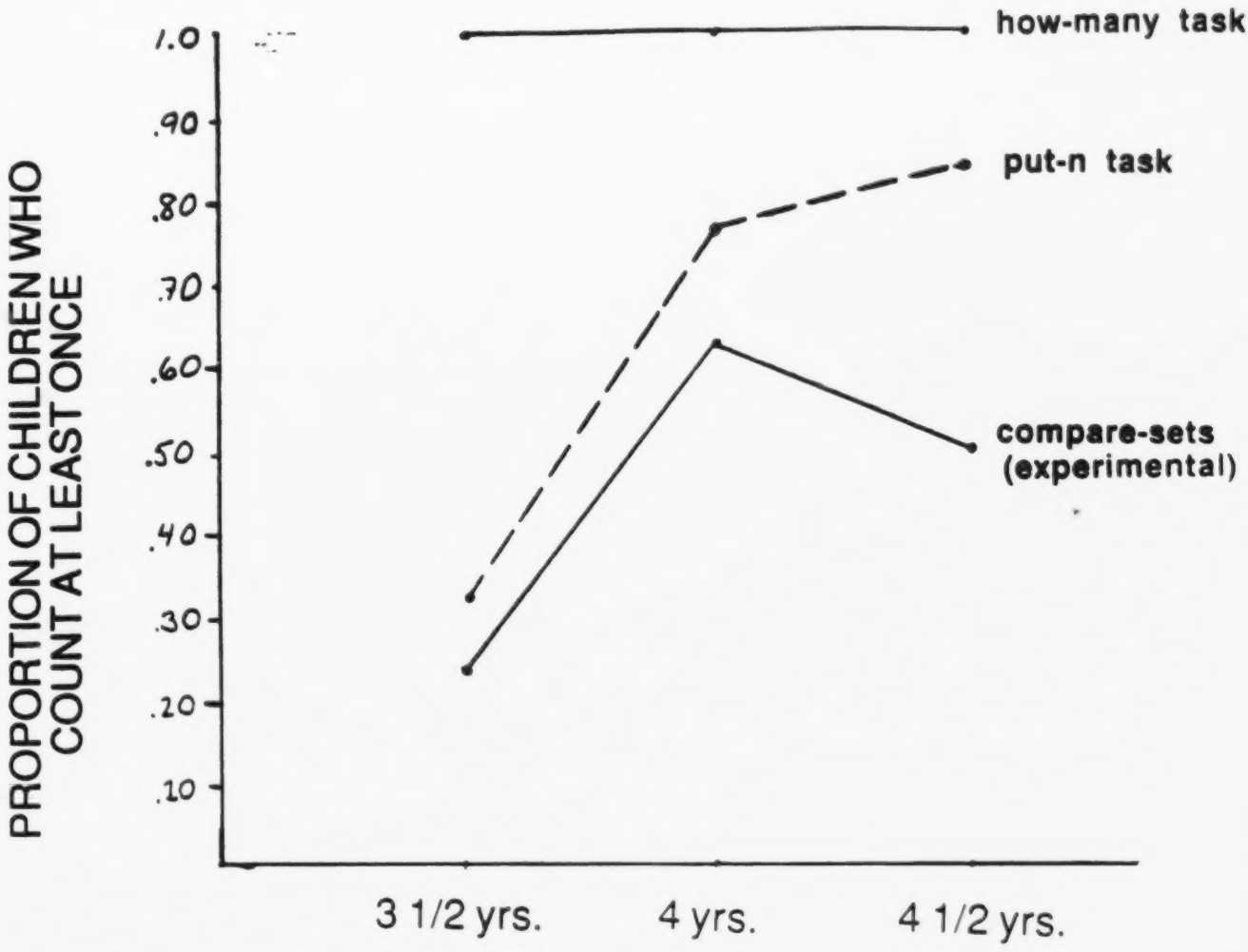
PROPORTION OF CHILDREN WHO COUNT AT LEAST ONCE
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
how-many task
put-n task
compare-sets (experimental)
3 1/2 yrs.
4 yrs.
4 1/2 yrs.

Figure 2

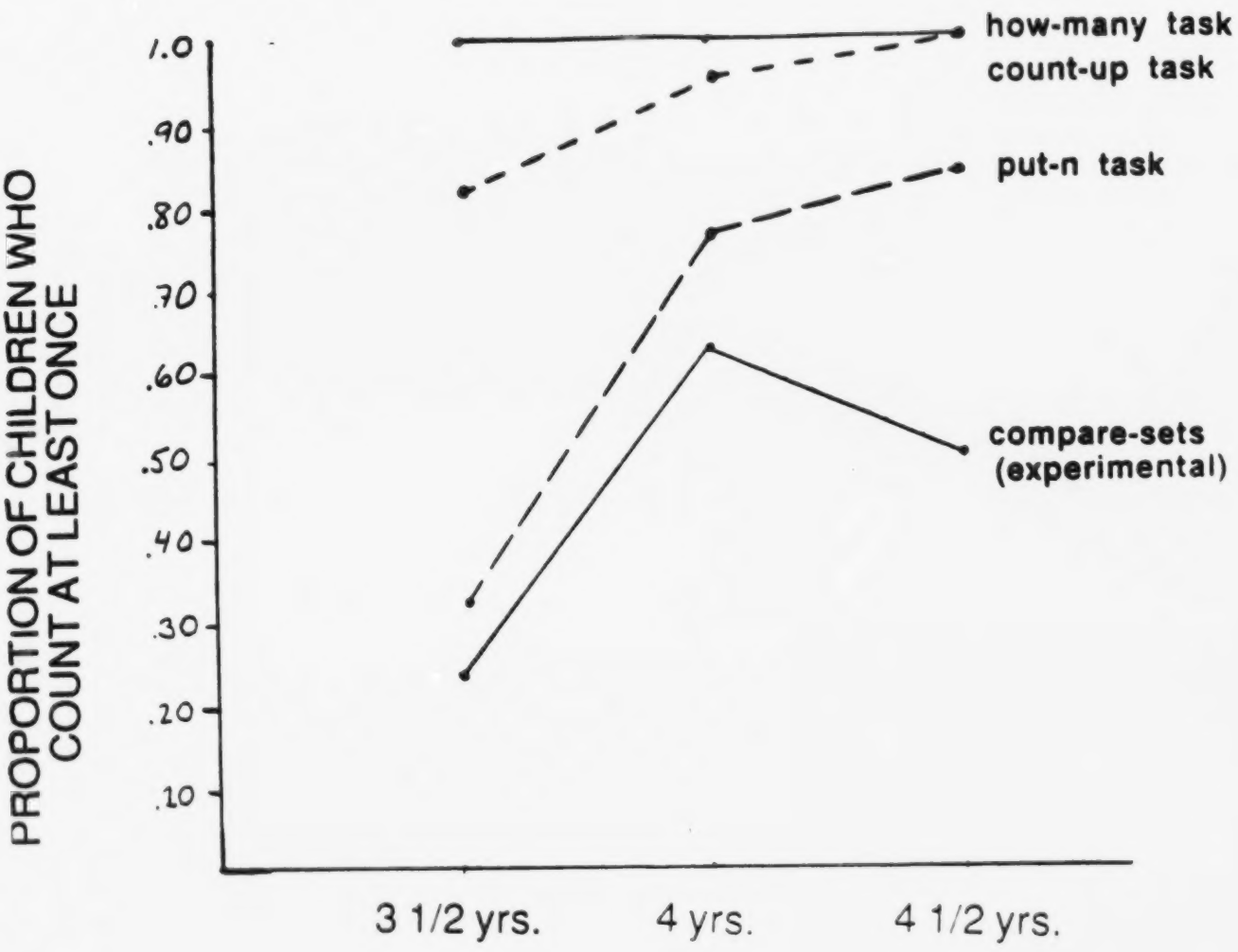
PROPORTION OF CHILDREN WHO COUNT AT LEAST ONCE
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
how-many task
count-up task
put-n task
compare-sets
(experimental)
3 1/2 yrs.
4 yrs.
4 1/2 yrs.

Figure 3

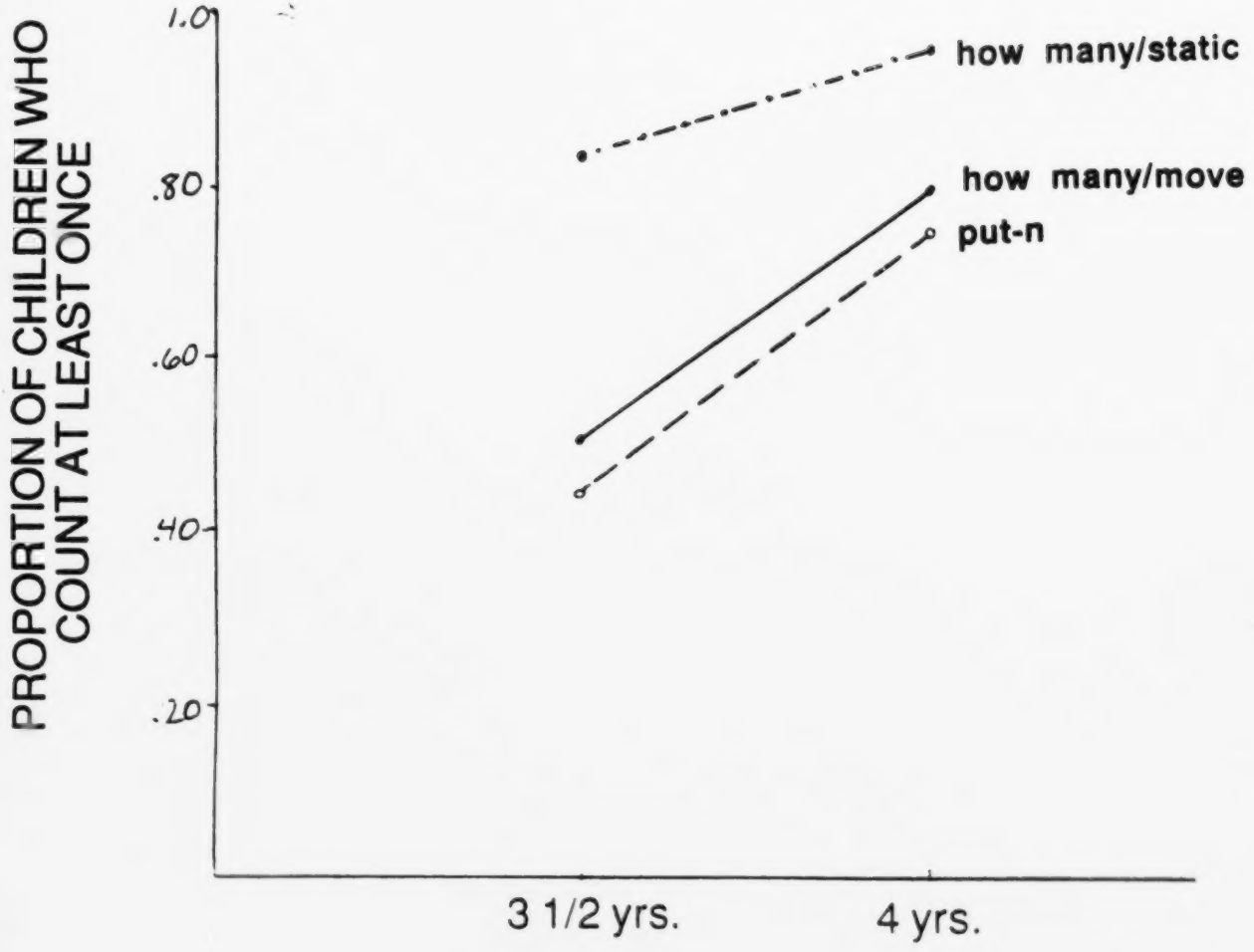
PROPORTION OF CHILDREN WHO COUNT AT LEAST ONCE
1.0
.80
.60
.40
.20
how many/static
how many/move
put-n
3 1/2 yrs.
4 yrs.

Figure 4